A Grasp Performance Criterion for Robot Hands Considering Multiple Aspects of Tasks and Hand Configurations

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Abstract—In this paper, a task-oriented grasp performance criterion for robot hands is discussed. The objectives of grasp criteria proposed in the past are stable grasping and the tasks that require adequate forces and moments on the grasped object. These criteria do not mainly consider the other physical quantities such as positions and velocities on the grasped object and the mechanical property of robot hands. We propose a grasp criterion for various kinds of tasks that evaluates the feasibility of the task with considering hand mechanisms and hand configurations. Our proposed method determine the grasping position by considering the efficiency of grasped object’s position, velocity, and force. As a result, this criterion decides the grasping position and the hand configuration. Additionally, we have shown that our proposed criterion can be applied to the design problem of robot hands.

I. INTRODUCTION

Grasping and manipulation of various objects are fundamental functions of robot hands. Recently, many multi-fingered robot hands have been developed[1][2][3][4]. In addition, robot hands whose specification is like human’s hands are also developed[5][6]. In the future, robot hands will be expected to deal with a large variety of objects and different kinds of tasks. One issue for these robot hands to realize dexterous grasping and manipulation like humans do is to decide the position to grasp. Selecting the grasping position affects “grasping and manipulation”. One approach to decide the grasping position for a robot is a grasp criterion. Accomplishing dexterous tasks needs a precise decision of the grasping position because the grasping position affects the feasible region of positions, velocities, and forces on the grasped object. Consequently, a grasp criterion will have an important role to deal with complicated tasks.

Various kinds of grasp criteria have been studied. One approach is using analysis based on geometry and dynamics. Many grasp criteria of this type have been proposed[7][8]. Chinellato et al. proposed the grasp criterion using the triangle which consists of finger tip positions[9]. Liegeois suggested an index using joint angles of finger to decide the best grasp[10]. Ferrari et al. proposed a grasp criterion whose index is the wrench space consisting of forces and torque which are applied to the grasped object[11]. This idea is applied to a grasp simulator of robot hands called GraspIt![14][16]. Some previous researches focused on tasks of robot hands. Li and Sastry developed a grasp criterion using the forces and torque which are applied to the object[15]. Li et al. proposed a grasp criterion for tasks with the requirement of forces on the object such as holding a pitcher to pour water from the bottle to the cup[16]. However, these criteria do not simultaneously consider other physical quantities such as positions and velocities on the grasped object, which are required to achieve dexterous tasks. However, these criteria do not consider the mechanical property of robot hands.

The other approach is using experimental results of subjects. Sugiyama et al. measured the grasping positions of subjects and discussed the relationship between the object shape and the grasping positions[17]. Ikeda et al. used the different length of the cylinder and evaluated the best length to be grasped by human[18]. Balasubramanian et al. investigated the method of grasping by measuring human’s grasping and improved the robustness of grasping[19]. Additionally, the objective of these previous grasp criteria is stable grasping. Therefore, the possibility about applying these criteria to skillful tasks is not mainly discussed.

In a previous paper[20], we have proposed a grasp criterion which considers multiple aspects of tasks: positions, velocities, and forces on the grasped object. Considering these physical quantities makes it possible to deal with skillful tasks that requires a finite region of positions and/or velocities. This criterion evaluates the feasibility of the task with comparing the desired region and the feasible region of these physical quantities at certain grasping positions. In addition, we have calculated the feasible region of physical quantities by considering the mechanism of robot hands. However, this criterion only evaluates grasping positions and hand configurations are not mainly considered as an evaluation index.

In this work, we consider hand configurations in addition to grasping positions and apply our proposed method to the design problem of robot hands. Furthermore, we improve the definition of the desired region of these physical quantities more intuitive and simple and made it possible to reduce calculation cost. We give some numerical examples of tasks in order to examine the validity of our proposed method. Additionally, we compare our proposed method with two criteria based on the wrench space.
II. PROPOSED GRASP CRITERION

We consider a robot hand having with multi-articulated fingers that grasps an object by its fingertips as shown in Fig. 1 (a). To determine a grasping position, we propose a grasp criterion considering multiple aspects of tasks and hand configurations. We define multiple aspects of tasks as the feasibility for position, velocity, and force on the grasped object.

A. Required Set and Feasible Set

In order to evaluate the feasibility of the task at certain grasping position, we use two kinds of set (Fig. 1 (a)). One is a required set which means the desired region of one physical quantity (position, velocity, or force) to achieve the task. The other set is a feasible set which indicates the realizable region of this physical quantity at certain grasping position and hand configuration. Inclusive relation between these two sets shows whether the task can be achieved at certain grasping position and hand configuration or not.

B. Evaluation Function

We propose a evaluation function that calculates an inclusive relationship between the required set and the feasible set. It means that this evaluation function shows the possibility for achieving the task. We define the evaluation function as a safety margin. The larger safety margin indicates the lower risk to fail the task. In other words, the optimum grasp has the largest safety margin among all the candidates and hand configuration.

Assume that \( l_f \) is the distance from the origin to the edge of the feasible set in the direction of cartesian coordinate in \( N \) dimension space (\( N = 3 \) for the 2 dimensional plane and \( N = 6 \) for the 3 dimensional space). and \( l_r \) is the distance from the origin to the edge of the required set in the same direction. Then, a safety margin \( a \) is defined by

\[
a = \begin{cases} 
\min \frac{l_f}{l_r} & \text{if } l_r \neq 0, \\
0 & \text{otherwise}.
\end{cases}
\]

The safety margin is the minimum value of \( l_f/l_r \) with respect to all possible directions. Let the safety margin of positions, velocities, and forces be \( a_p, a_v, \) and \( a_f \) at certain grasping
position, respectively. We use \( a_{gm} \) as an index with the following definition:

\[
a_{gm} = \min \{ a_g, a_v, a_f \}. \tag{2}
\]

The minimum safety margins which have the highest risk for the task are evaluated. Note that, intuitively speaking, the position is related to the workspace evaluation, the velocity is related to the manipulability analysis, and the force is related to the grasping force analysis.

III. CALCULATION METHOD FOR REQUIRED SET AND FEASIBLE SET

A. Analytic Model

We consider the model that the robot hand with \( n \) fingers grasp the object (Fig. 1 (c)). Additionally, we define \( \mathbf{r}_O = (x_O, y_O, z_O, \phi_O, \psi_O) \) as the position vector consisting of position at centroid \((x_O, y_O, z_O)\) and posture of the object \((\phi_O, \psi_O)\), \( \mathbf{\theta}_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{im}]^T \in \mathbb{R}^m \) as the joint angle vector on ith finger, \( \mathbf{r} = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{R}^{n \times n} \) as the joint angle velocity vector of all fingers, \( \mathbf{\tau}_i = [\tau_{i1}, \tau_{i2}, \ldots, \tau_{in}]^T \in \mathbb{R}^m \) as the joint torque vector on ith finger, \( \mathbf{\tau} = [\tau_1, \tau_2, \ldots, \tau_n]^T \in \mathbb{R}^{m \times n} \) as the joint torque vector on all fingers.

The followings are the conditions which are also considered:

- The fingers are solid and make frictional point contacts with the object. Therefore, the contact points between fingertips and the object can be regarded as free revolute joints.
- All fingers do not intersect.
- Gravitational force does not work on them.

B. Required Set

The required set is specified by the task requirement. An example of the required set is shown in Fig. 2. If the object is required to move within a limited range of \( x, y, \phi \) axis, the required position set is given by a cube (Fig. 2 (a)). If the task requires unilateral velocity on \( \phi \) axis, the required velocity set is designed to be a line along \( \phi \) axis (Fig. 2 (b)). In order to resist the forces which has a fixed magnitude but might be applied from all the directions, the required force set should be the circle on \( F_xF_y \) plane (Fig. 2 (c)).

C. Feasible Set

1) Feasible Position Set: The feasible position set is the realizable region of the position vector \( \mathbf{r}_O \) and is calculated by using inverse kinematics. The inverse kinematic of this model is obtained by

\[
\mathbf{\theta} = \mathbf{f}_{inv}(\mathbf{r}_O) \tag{3}
\]

Note that \( \mathbf{f}_{inv}(\cdot) \) denotes an inverse kinematics.

The condition is a range of joint angles calculated in order to avoid singular configurations. Hence, the range of joint angles is set to \( 0 \leq \theta_{ij} \leq \pi \). Therefore the feasible position set is defined as follows:

\[
S_{FP} = \{ \mathbf{r} \mid \mathbf{\theta} = \mathbf{f}_{inv}(\mathbf{r}_O), 0 \leq \theta_{ij} \leq \pi \}, \quad i \in [0, n], j \in [0, m]. \tag{4}
\]

2) Feasible Velocity Set: A feasible velocity set denotes the realizable region of the velocity vector \( \dot{\mathbf{r}}_O = (\dot{x}_O, \dot{y}_O, \dot{z}_O, \dot{\phi}_O, \dot{\psi}_O) \).

Let us solve \( \dot{\mathbf{r}}_O \) from joint angular velocities of fingers. Assume that \( \mathbf{\theta}_i \) is the angular velocity at finger \( i \), we can solve the equation of fingertips by using two types of expression. One route is using forward kinematics of finger \( i \) and the other route is using \( \mathbf{r}_O \). Given that \( \mathbf{p}_{fi} \) is the position of the fingertip \( i \), the velocity of \( \mathbf{p}_{fi} \) is obtained by

\[
\dot{\mathbf{p}}_{fi} = \frac{d}{dt} g_{ki}(\mathbf{\theta}_i) = \mathbf{J}_E \dot{\mathbf{\theta}}_i, \tag{5}
\]

\[
\dot{\mathbf{p}}_{fi} = \frac{d}{dt} g_{ai}(\mathbf{r}_O) = \mathbf{J}_G \dot{\mathbf{r}} \tag{6}
\]

where \( g_{ki}(\mathbf{\theta}_i) \) is the kinematics of finger \( i \), \( g_{ai}(\mathbf{r}_O) \) is position vector \( \mathbf{p}_{fi} \) in terms of \( \mathbf{r}_O \). \( \mathbf{J}_E \) is the Jacobian matrix at finger \( i \), \( \mathbf{J}_G \) is the grasp matrix at finger \( i \). From (5) and (6), we obtain

\[
\mathbf{J}_G \dot{\mathbf{r}} = \mathbf{J}_E \dot{\mathbf{\theta}}_i. \tag{7}
\]

Therefore, we have

\[
\mathbf{J}_G \dot{\mathbf{r}} = \mathbf{J}_E \dot{\mathbf{\theta}} \tag{8}
\]

where

\[
\mathbf{J}_E = \begin{bmatrix}
J_{E1} & \mathbf{O} \\
J_{E2} & \ddots \\
\mathbf{O} & \ddots & \ddots \\
\end{bmatrix} \in \mathbb{R}^{2n \times 2n},
\]

\[
\mathbf{J}_G = \begin{bmatrix}
J_{G1} \\
J_{G2} \\
\vdots \\
J_{Gn}
\end{bmatrix} \in \mathbb{R}^{2n \times 6},
\]

\[
\mathbf{\theta} = \begin{bmatrix}
\theta_1 \\
\theta_2 \\
\vdots \\
\theta_n
\end{bmatrix} \in \mathbb{R}^{mn}.
\]

Aside from singular configurations, \( \mathbf{J}_E \) has an inverse matrix. Therefore, joint angular velocity vector \( \dot{\mathbf{\theta}} \) is represented by

\[
\dot{\mathbf{\theta}} = \mathbf{J}_E^{-1} \mathbf{J}_G \dot{\mathbf{r}}. \tag{9}
\]

Note that we assume that robotic hand does not have redundancy.

The condition of the feasible velocity set is a range of joint angular velocities. We define \( \theta_{max} \) as the maximum absolute value of a joint angular velocity. In this paper, \( \theta_{max} \) is set to be 1. Hence, the feasible velocity set is obtained by

\[
S_{Fv} = \{ \dot{\mathbf{r}} \mid \dot{\mathbf{\theta}} = \mathbf{J}_E \mathbf{J}_G \mathbf{\theta}, |\dot{\theta}_{ij}| \leq 1 \}, \quad i \in [0, n], j \in [0, m]. \tag{10}
\]
where

\[ \mathbf{F}_O = \left( \mathbf{f}_O, \mathbf{n}_O \right)^{T} \]

is the force of the object consisting of force at the center of the mass \( \mathbf{f}_O = (f_{Ox}, f_{Oy}, f_{Oz})^{T} \) and moment of the object \( \mathbf{n}_O = (n_{Ox}, n_{Oy}, n_{Oz})^{T} \).

Therefore, the feasible force set is defined as

\[ S_{Ff} = \left\{ \mathbf{F}_O \mid \mathbf{F}_O = \mathbf{Gf}, \mathbf{a}_i \cdot \mathbf{f}_i \geq 0, \right\} \]

where \( \mathbf{a}_i \cdot \mathbf{f}_i \geq 0 \) is the condition of grasping force. Forces should be inside the friction cone in order to utilize the friction. Assumed that \( \mu \) is coefficient of static friction, the condition is given by

\[ \frac{\mathbf{a}_i \cdot \mathbf{f}_i}{\left\| \mathbf{f}_i \right\|} > \frac{1}{\sqrt{1 + \mu^2}} \]

Therefore, feasible force set is defined as

\[ S_{Ff} = \left\{ \mathbf{F}_O \mid \mathbf{F}_O = \mathbf{Gf}, \mathbf{a}_i \cdot \mathbf{f}_i \geq 0, \right\} \]

\[ \mathbf{a}_i \cdot \mathbf{f}_i \geq 0, \quad i \in [0, n], j \in [0, m]. \] (19)

\[ \text{D. Calculation Algorithm} \]

The best grasping positions and the hand configuration for tasks are decided by using following steps:

1. Step1 Calculate each safety margin \( (a_{ij}, a_{ij}, a_{ij}) \) of grasping position candidates and the hand configuration
2. Step2 Select minimum safety margin \( a_{gmin} \)
3. Step3 Select maximum \( a_{gmin} \) from all grasping position candidates and hand configurations

If the grasping positions and the hand configuration are not decided because there are multiple minimum safety margins, repeat Step2 – Step3 by using the second smallest value and the third smallest value.

\[ \text{IV. Numerical Example} \]

\[ \text{A. Task specifications} \]

In this paper, we examine tasks on 2 dimensional plane with two-fingered robotic hand. We examine the validity of our proposed method with the following tasks:

- Pulling the object (Fig. 4)
- Drawing lines (Fig. 5)
- Opening/Closing a bottle cap (Fig. 6)

The model and grasping positions candidate is shown in Fig. 4 - Fig. 6. Task requirements and parameters are listed in TABLE I. In our previous work[20], we only evaluated grasping positions. Here, we also evaluate distance \( d \) and posture \( \phi \) as it is shown in Fig. 7.

\[ \text{B. Calculation results} \]

Calculation results of each task are shown in TABLE II, Fig. 8. If the object and the hand intersects, the safety margins are set to be 0. According to our proposed method, the best grasp of these tasks are the followings:

- Pulling the object: Position A, \( d = 1.0, \phi = 0.0 \) [rad]
- Drawing lines with a pen: Position E, \( d = 1.0, \phi = 0.0 \) [rad]
- Opening/Closing a bottle cap: Position B, \( d = 1.0 \)

As it is shown in the safety margins \( a_{gmin} \) in TABLE II, our proposed method can be used to decide the hand configuration and the posture.
C. Discussion

According to the result of Example 1 (Fig. 8 (a)), our proposed method selects the grasping position A which can guarantee comparatively larger workspace than other grasping positions B – H. Thus, this grasp criterion could decide the grasping position for the task with the requirement of the region of positions. This is because our proposed method consider the positions, velocities, and forces on the grasped object. This aspect is one of the advantages of our criterion compared with the previous grasping criterion which focuses on stable grasping and the tasks with the requirement of forces on the grasped object.

The result of Example 2 (Fig. 8 (c)) indicates that our proposed method can deal with tasks with the requirement of forces on the grasped object similar to the grasp criteria proposed in the past. We found that the evaluation function
Intersection between the object and the hand. Additionally, safety margins \( a_{gm} \) in TABLE II differ if \( d \) and \( \phi \) is different. Especially, \( a_{gm} \) of \( \phi = 0 \) [rad] shows higher value than others. It is because we consider the intersection of the object with robotic hand.

Table II

<table>
<thead>
<tr>
<th>Task</th>
<th>Distance ( d )</th>
<th>Posture ( \phi )</th>
<th>Safety margin ( a_{gm} )</th>
<th>Position candidates</th>
<th>Selected safety margin</th>
<th>Selected position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulling the object</td>
<td>( d = 0.5 )</td>
<td>( \phi = 0 ) [rad]</td>
<td>( a_{p} ) (1.60 [-])</td>
<td>A</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 0.5 )</td>
<td>( \phi = \pi/4 ) [rad]</td>
<td>( a_{p} ) (0.62 [-])</td>
<td>A – B</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 1.0 )</td>
<td>( \phi = 0 ) [rad]</td>
<td>( a_{p} ) (1.71 [-])</td>
<td>A – C</td>
<td>( a_{p} ) (3.60 [-], 3rd lowest)</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>( d = 1.0 )</td>
<td>( \phi = \pi/4 ) [rad]</td>
<td>( a_{p} ) (1.59 [-])</td>
<td>A – D</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 1.5 )</td>
<td>( \phi = 0 ) [rad]</td>
<td>( a_{p} ) (1.29 [-])</td>
<td>A – E</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 1.5 )</td>
<td>( \phi = \pi/4 ) [rad]</td>
<td>( a_{p} ) (1.05 [-])</td>
<td>A – E</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Drawing lines with a pen</td>
<td>( d = 0.5 )</td>
<td>( \phi = 0 ) [rad]</td>
<td>( a_{p} ) (1.20 [-])</td>
<td>E</td>
<td>( a_{p} ) (3.60 [-], 1st lowest)</td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>( d = 0.5 )</td>
<td>( \phi = \pi/4 ) [rad]</td>
<td>( a_{p} ) (0.88 [-])</td>
<td>C</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 0.5 )</td>
<td>( \phi = \pi/2 ) [rad]</td>
<td>( a_{p} ) (1.05 [-])</td>
<td>C</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 1.0 )</td>
<td>( \phi = 0 ) [rad]</td>
<td>( a_{p} ) (0.95 [-])</td>
<td>E</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 1.5 )</td>
<td>( \phi = \pi/4 ) [rad]</td>
<td>( a_{p} ) (0.29 [-])</td>
<td>E</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 1.5 )</td>
<td>( \phi = \pi/2 ) [rad]</td>
<td>( a_{p} ) (0.52 [-])</td>
<td>A</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( d = 1.0 )</td>
<td>-</td>
<td>( a_{p} ) (1.04 [-])</td>
<td>B</td>
<td>( a_{p} ) (1.04 [-], 1st lowest)</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>( d = 1.5 )</td>
<td>-</td>
<td>( a_{p} ) (0.60 [-])</td>
<td>B</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 8. Numerical calculation results of the task.

Numerical result of Example 3 (Fig. 8 (e)) shows that the safety margin of position at candidates A – C is higher than that of candidates D – F. This is because we consider the intersection between the object and the hand. Additionally, the safety margin of force at candidate B is higher than that of candidates A and C. It is because of considering the friction cones and this result coincided with the intuition of the human.
V. COMPARISON WITH THE GRASP CRITERION BASED ON WRENCH SPACE

A. Grasp Criterion Based on Wrench Space

We compare our proposed method with the grasp criterion used in GraspIt![14]. The index of this criterion is the volume of wrench space and the maximum radius of the inscribed ball in it. We define the index of the volume, maximum radius of the inscribed ball as $E_v$, $r_v$, respectively. Assume that the wrench at finger $i$ is $w_i$, the wrench space is $P$, $w_i$ and $P$ are defined by

$$ w_i = \left[ \begin{array}{c} f_i \end{array} \right] = \sum_{j=1}^{s} \alpha_{i,j} w_{i,j}, $$

$$ \alpha_{i,j} \geq 0, \quad \sum_{i=1}^{n} \sum_{j=1}^{s} \alpha_{i,j} \leq 1 $$ (20)

$$ P = \text{ConvexHull} \left( \bigcup_{i=1}^{n} (w_{i,1}, \cdots, w_{i,m}) \right). $$ (21)

Note that $f_i$ is the finger tip force at the corner $j$ and finger $i$, $r_i$ is the vector from the center of mass to the finger tip $i$, $w_{i,j}$ is a wrench at the corner $j$ of finger $i$, $\alpha_{i,j}$ is the weight coefficient at the corner $j$ of finger $i$, and “ConvexHull” denotes convex set. $w_i$ is the sum of $w_{i,j}$ created by approximating friction cone in Fig. 9. Corner $j$ is created by the friction cone which is approximated as polyhedron.

B. Calculation results

We used the task: “Stable grasping” and conditions are set to be $d = 1.0, \phi = 0.0$ [rad]. Results are shown in Fig. 10 and the best grasp are determined as followings:

- Proposed method: Position A
- Wrench space volume ($E_v$): Position F, G, and H
- Maximum radius of inscribed ball ($r_v$): Position G

C. Discussion

The evaluation result of the wrench space shows that the best grasp is grasping the longer part of the object because these criteria are mainly considering the moments on the object. It means that these criteria have possibility to select the grasping positions that is too wide for robot hands to grasp. On the other hand, the result of our proposed method means that the best grasp is grasping the longer part of the object because we consider hand mechanisms. These results show the validity of taking hand mechanisms into consideration.

VI. A DESIGNING METHOD CONSIDERING MULTIPLE ASPECTS OF TASKS AND HAND CONFIGURATIONS

A. Design guide

In order to deal with the specific task, we applied our proposed method to design the robot hand. We designed the structure of the robot hand, actuator joint angular velocity, and actuator joint torque.

B. Calculation results

We designed the hand mechanism of the hand for the task: “Pulling the object” by using the candidates in Fig. 11 (a) – (e) and TABLE III. Results are shown in Fig. 11 (f) – (j) and the best design is Design 1 and pattern A.
C. Discussion

The safety margin of position is higher if the ratio of the links is 1:1. It seems that the condition of the intersection between the link and the bottom of the hand affected the result. In addition, the safety margin of velocity is higher if the mechanism is symmetric. This is because the velocity of the object which is easier to apply is different if the mechanism is not symmetric. From these result, our proposed method can be applied to the designing method of the hand because safety margins differ if the hand mechanism and the specification of actuator changes.

VII. Conclusions and Future Works

We proposed a grasp criterion which considers multiple aspects of tasks such as positions, velocities, and forces on the grasped object. Evaluation of the feasibility of the task is done by comparing two kinds of set: a required set and a feasible set. Inclusive relationships of these sets shows the feasibility of the task. The numerical calculation results of the example indicate that our grasp criterion determine the grasping position and the hand configuration. In the future, we are planning to consider the dynamics to calculate feasible velocity sets and feasible force sets. Constructing the method of evaluating on continuous candidates is also future work of our research.

REFERENCES